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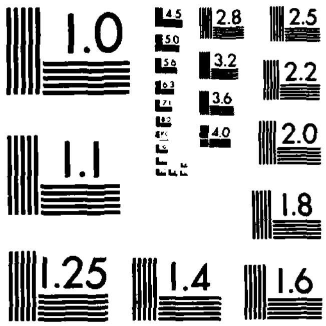

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THE INDIVIDUAL FREEDOM ALLOWED BY THE VALUE RESTRICTION CONDITION

by

HERVÉ RAYNAUD

TECHNICAL REPORT NO. 360
January 1982

A REPORT OF THE
CENTER FOR RESEARCH ON ORGANIZATIONAL EFFICIENCY
STANFORD UNIVERSITY
Contract ONR-N00014-79-C-0685, United States Office of Naval Research

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THE INDIVIDUAL FREEDOM ALLOWED BY THE VALUE RESTRICTION CONDITION*

by

Hervé Raynaud

1. Introduction

It is known that the majority method (for an odd N, a is before b in the collective order iff more individual orders rank a before b than b before a) does not always yield a total order.

The frequency of the so-called "Condorcet effect" has been extensively derived and computed. (Guilbaud [1968], and Fishburn, Gehrlein, Maskin [1979] are the earliest and the latest papers I know on the subject).

K. Arrow, following the proof of ~~his celebrated~~ impossibility theorem, ~~(Arrow [1963]) has given~~ an example of possible escape by some "natural" restrictions on the domain of the individual orders is given. Since that time, various other conditions have been proposed (see, for instance, Köhler [1978], Romero [1978], Ward [1961], Sen [1979]). Then there began to appear some sufficient and necessary "in a certain sense" conditions. The value restriction condition is among them. It seems to have a particular psychological interpretation which can justify a special treatment. In the case of total orders, the value restriction condition is equivalent to Ward's condition [1961] and can be written in the three following equivalent forms:

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*This research was supported by the Office of Naval Research Grant ONR N00014-7-C-0685 at the Center for Research on Organizational Efficiency, Stanford University. This research has been done with the help of Stanford University, the Fulbright Exchange Program and the University of Grenoble.

1. There is no $Y \subset X$, $|Y| \geq 3$, and no subset $(\theta_{i_1},\ldots,\theta_{i_{|Y|}})$ such that $\theta_{i_1}(Y),\ldots,\theta_{i_{|Y|}}(Y)$ form the lines of a circulant matrix.
2. There is no Condorcet triple, i.e. there are no three objects (say, a,b,c) and no three individual orders such that their restrictions to the three considered objects constitute a cyclic triple (abc, bca, cab or bac, acb, cba).
3. For every triple of objects T, there is an object, say x, and a rank $j \in \{1,2,3\}$ such that in any $\theta_i(T)$, x is never ranked the j-th of the three objects in T.

This condition is necessary in the sense that it ensures the transitivity of the majority method for any odd subset of individual votes. Psychologically speaking one can understand that any profile with a cyclic triple can generate endless discussions between three voters, and endless hesitations when the voters are criteria in a decision making problem.

The third form will be used to study the condition, because it has appeared to be more tractable than the others.

It is well known that when the number of objects increases, the frequency of cyclic triples increases very quickly, which means that satisfying the condition "per chance" becomes very unlikely.

The point of view retained here for enumeration is different. It focuses on the maximal number of different votes an individual can express. In other words, one will count the maximum number of different individual orders that can be found in a profile satisfying the condition.

Let $M(n)$ be this number. We already know that $M(n) \geq 2^{n-1}$ from Raynaud [1981]. I have proved in this quoted paper that the maximum number of different votes in profiles following Inada's conditions [1964] are 2^{n-1}, and Inada's conditions are particular cases of the value restriction condition.

Mathematicians have been, as far as I know, unsuccessful in the search for $M(n)$ which reveals to be a shrewd combinatorial problem. Exploring $M(4)$ as a first non-trivial problem, I met some generalizable lemmas and the result which explains the shrewdness of the general problem. $M(4) = 9$ and the critical sets of permutations are very rare (24 over the $\simeq 5 \times 10^{11}$ possible profiles made of nine different votes on four objects!).

Throughout the paper, geometrical arguments observed on the permutohedron (Guilbaud [1960]) are used to help the reader. Exhibition of symmetries and isomorphisms will allow many shortcuts in the proofs (easily algebraicized if necessary) and, I hope, an easier reading of this note.

Figure 1 represents the permutohedron on the four objects a, b, c, d. The vertices of the permutohedron on n objects are the permutations on n objects. Two vertices are joined by an edge iff the two corresponding permutations can be obtained, one from the other, by transposition of two neighbor objects.

It has been proven that this graph is a convex polyhedron in $\mathbb{R}^{n-1}$. The structure of its facets and their adjacency properties are well known.

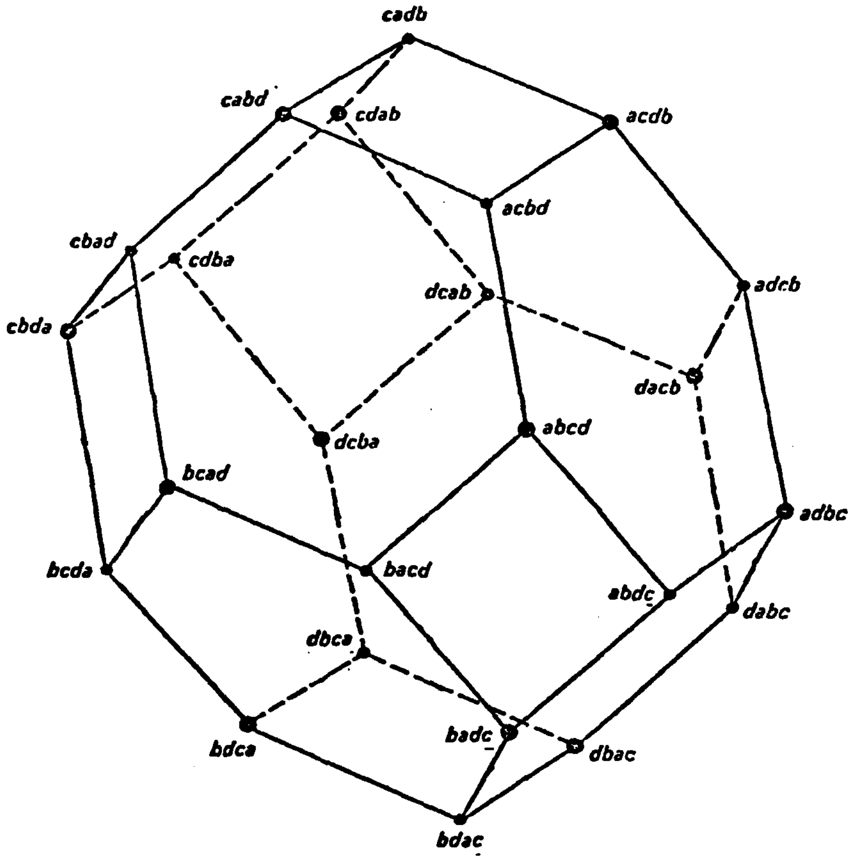
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Figure 1

Preliminary Remarks

-In what follows, $X = \{a,b,c,d\}$, and the considered permutohedron will be the permutohedron with $X = \{a,b,c,d\}$. All profiles will follow the value restriction condition.

-There are four unordered triples on four objects, and the value restriction conditions can be decomposed into four conditions, each one concerning one of the four triples.

-To respect one of these conditions means to prohibit one third of the vertices of the permutohedron - as it prohibits one third of the possible permutations of the considered triple which are equally frequent among the 24 permutations on four objects.

-However, on the permutohedron, the "shapes" of the conditions "x is never second" on one hand and "x is never first" or "x is never last" on the other hand, are very different.

Without loss of generality, one can:

-take the triple $\{a,b,c\}$ for example;

-take $x = a$;

Then one can represent the three conditions by deletion of the edges of the permutohedron adjacent to the forbidden permutations.

Figure 2 represents "a never second in {a,b,c}" (which will be denoted a n 2/b c).

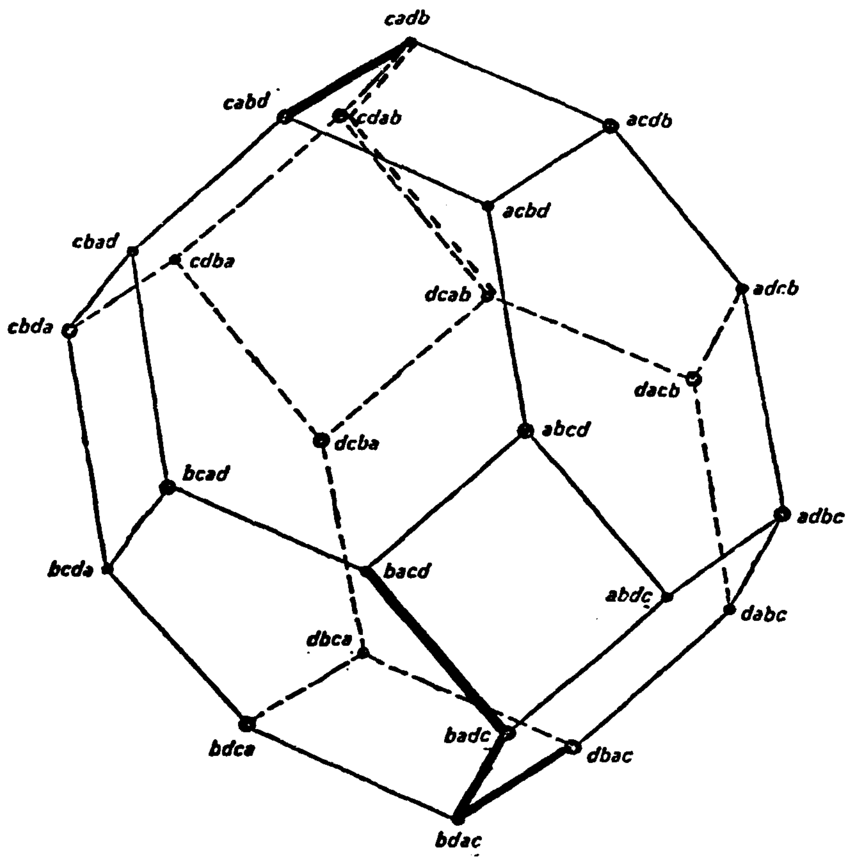

Figure 2

Figure 3 represents "a never the first and a never the last in {a,b,c}" (which will be denoted a n 1/b c and a n 3/b c).

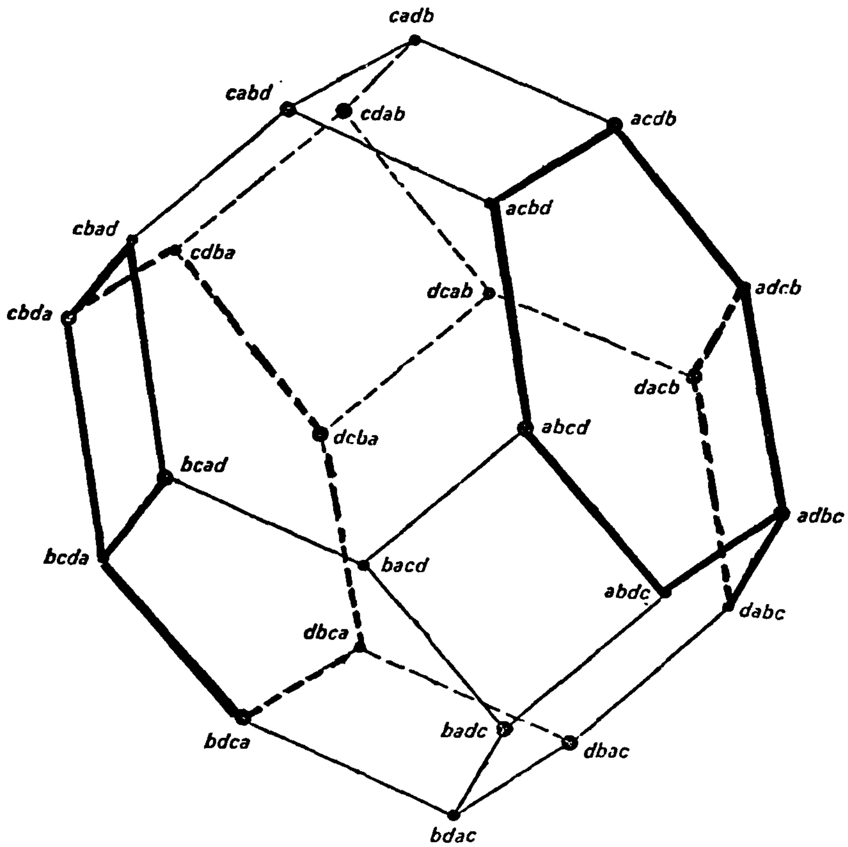

Figure 3

-The three sets of permutations are disjoint. What is more, the set corresponding to a n 2/b c disonnects the permutohedron. If one considers the sets of prohibited permutations for the conditions a n 2/b d, a n 2/b c, a n 2/c d one can see that they cover a circuit of the graph, as represented in Figure 4; each vertex of this circuit is included in two of these conditions.

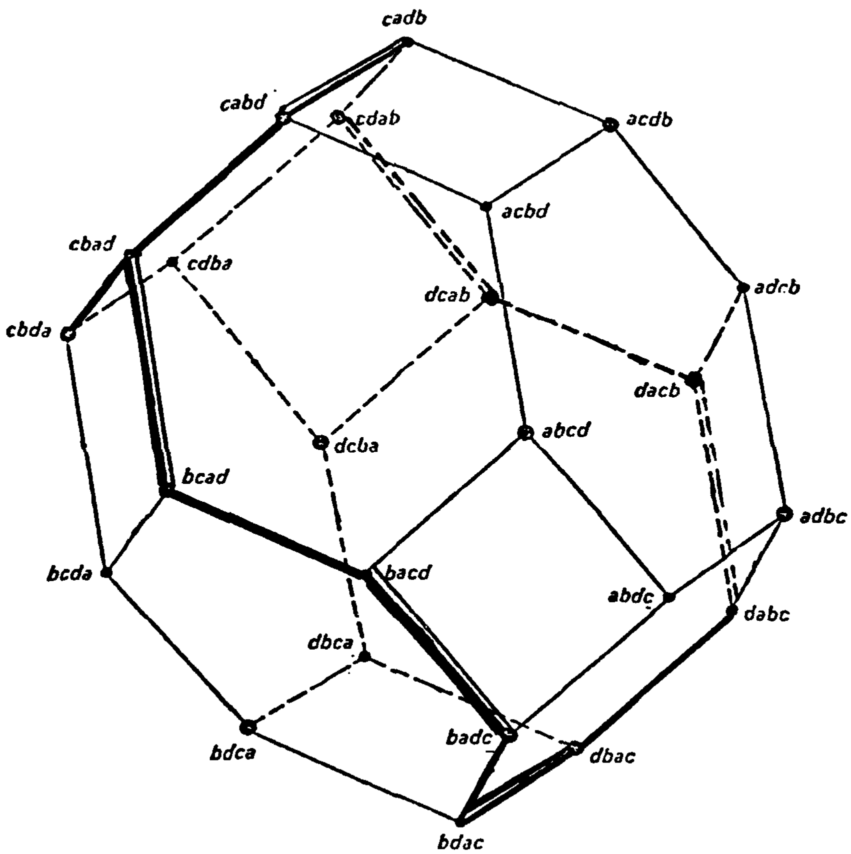

Figure 4

-The two other conditions will be called extremals. Intuitively, they consist in the prohibition of one hexagon and one of its adjacent squares. If one considers, conversely, one hexagon and one of its adjacent squares, the hexagon will indicate "the object x is never 1st or 3rd" and the square will make precise on which triple the condition holds.

-If a profile counts strictly more than $2^3 = 8$ different individual orders and follows the value restriction condition, it will be called a candidate. Clearly, a candidate cannot respect two conditions on the same triple because it would leave only (from the previous remark) eight non-prohibited vertices on the permutohedron.

-A condition will be considered of type i if it says that some object cannot take the i-th rank.

I. <u>Proposition 1</u>: In any candidate profile,

1. the objects never in second are all different,
2. they are different from the objects following extremal conditions,
3. if x is "never first" for one triple, it cannot be "never last" for another one.

<u>Lemma 1</u>: If the four conditions followed by a profile are of the same type, then this profile is not a candidate.

<u>Proof</u>: The profile then would follow one of Inada's conditions and according to my previous result (Raynaud [1981]) would not be a candidate.

<u>Lemma 2</u>: Let x and y follow conditions of type 2 for the same profile E (following the value restriction condition). If E is a candidate, then $x \neq y$.

<u>Proof</u>: Suppose, on the contrary, that (without loss of generality) "a" follows two conditions of type two. It can be checked (cf. Figure 4), that all 12 vertices on the circuit are cancelled. Then, it is natural to look at the conditions which are possible for $\{b,c,d\}$.

In fact, the cancellation of the 12 vertices of the circuit acts exactly as if "a" were following the three possible conditions of type two. Hence we know that if E is a candidate, necessarily, the condition on $\{b,c,d\}$ will be extremal.

Because of the total symmetry of the problem in b, c and d, we can choose the case b n 1/c d. One can see on Figure 5, that it adds the four underlined vertices to the previous set of prohibited vertices. Their total number then reaches 16 which is contradicting with the fact that E is supposed to be a candidate. For symmetry seasons, the same would occur with the case b n 3/c d.

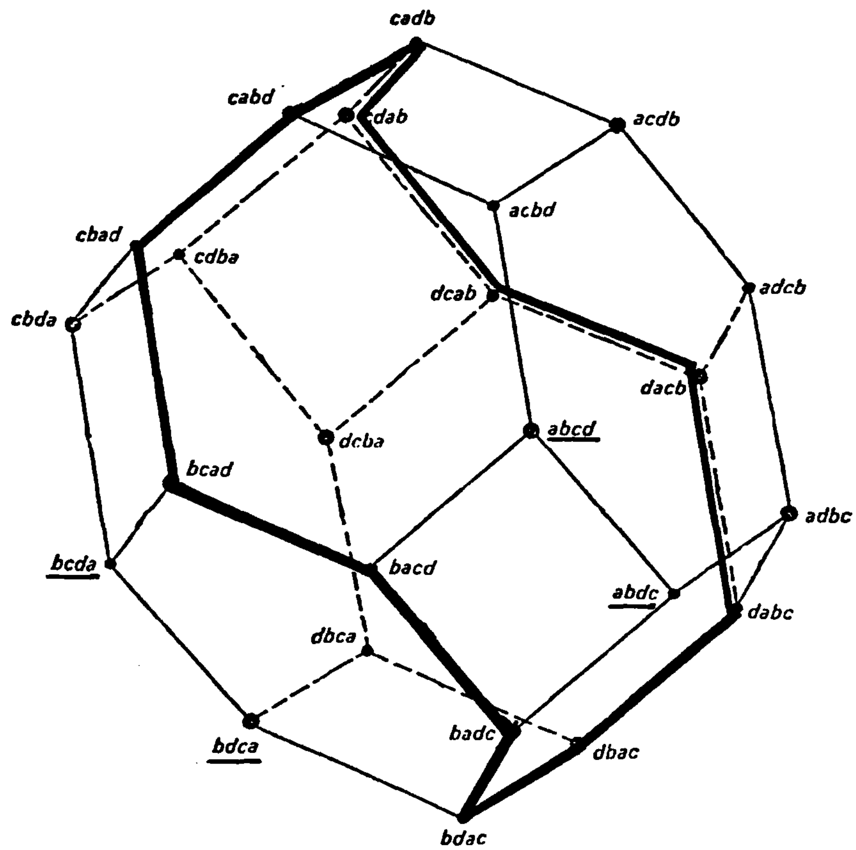

Figure 5

Lemma 3: One object cannot follow a condition of type 1 and a condition of type 3 (in a candidate profile).

Proof: The two sets of corresponding prohibited permutations are clearly distinct (one object cannot be the first and the last of a triple in the same permutation) and each one counts eight vertices.

Lemma 4: In a candidate profile, there is no object satisfying a condition of type 2 and an extremal condition.

Proof: Without loss of generality one can consider any one of the conditions a n 3 and any one of the conditions a n 2.

From Figure 4 and Figure 6, the two corresponding sets of prohibited permutations intersect in at most two vertices (which makes 14 prohibited permutations in the two cases), and in fact always in two vertices because the only condition of type 2 on a which has no prohibited vertex in common with a n 3/b c is precisely a n 2/b c.

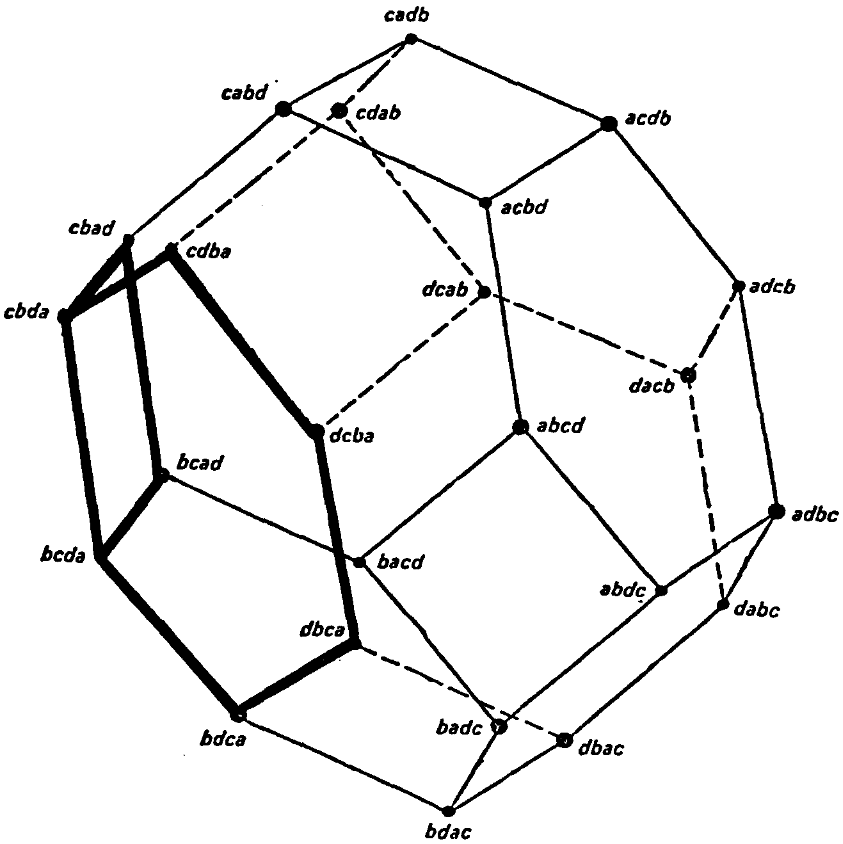

Figure 6

As shown clearly on Figure 7 (which summarizes the prohibited vertices by conditions a n 3/b c and a n 2/b d) only two more hexagons can be used without deleting more than one vertex -- namely, the hexagon where b is first and the hexagon where d is first. However, for each of these, the only adjacent squares already entirely prohibited imply that the conditions would have to be b or d n 1/{a,b,d} -- which is not possible.

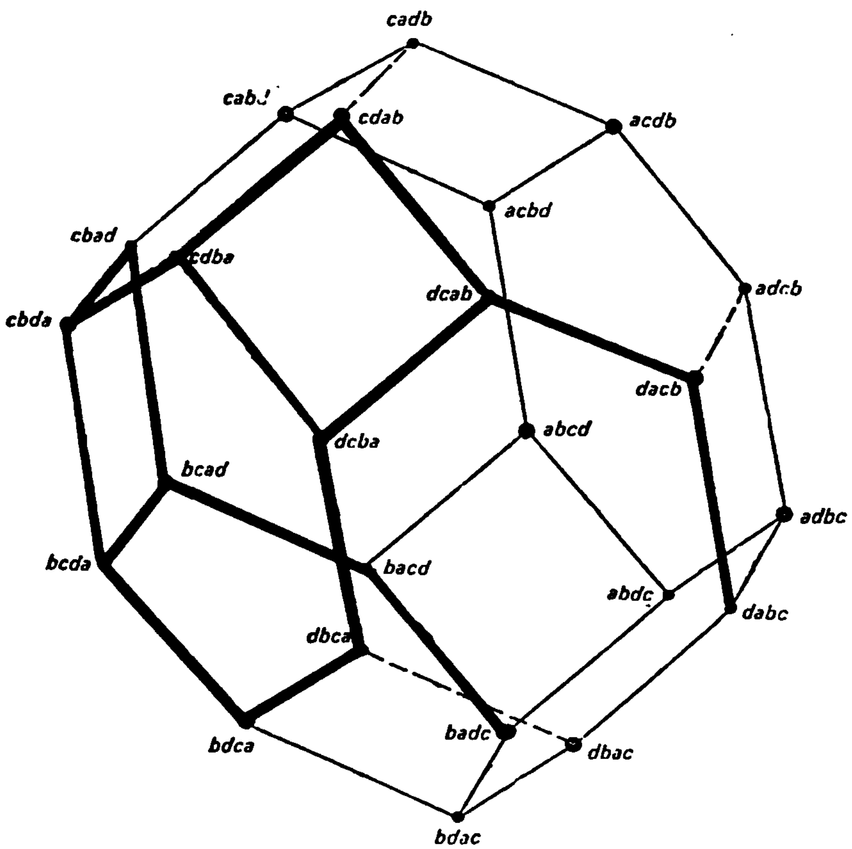

Figure 7

Hence the two other conditions on triples $\{a,c,d\}$ and $\{b,c,d\}$ have to be of type 2. But any condition of type 2 on $\{b,c,d\}$ will include two vertices among the not yet prohibited vertices of the hexagon where a is first, and the profile will not be a candidate.

The summary of these last three lemmas gives Proposition 1. In effect:

1. If no object follows an extremal condition and a condition of type 2, it means that the objects following a pair of such conditions have to be different (Lemma 4).
2. If no object follows a condition of type 1 and a condition of type 3, it means that the set of objects following conditions of type 3 is disjoint from the set of objects following conditions of type 1 (Lemma 3).
3. The assertion of Lemma 2 remains identical.

II. The object of this section is to prove that there is no candidate profile with conditions of type 2.

Corollary 1: There is no candidate profile with exactly one "not in the middle" condition (or with exactly three extremal conditions).

Proof: The extremal conditions can demand, a priori, 3, 2 or 1 hexagons.

The case of three hexagons is easy. In order not to reach 18 prohibited vertices, at least two are adjacent. But if two are adjacent, then the third one will be either adjacent or opposite to one of the two first ones. Hence it is necessarily adjacent.

Without loss of generality we can consider the three adjacent hexagons of Figure 8. They, necessarily, belong to conditions on different

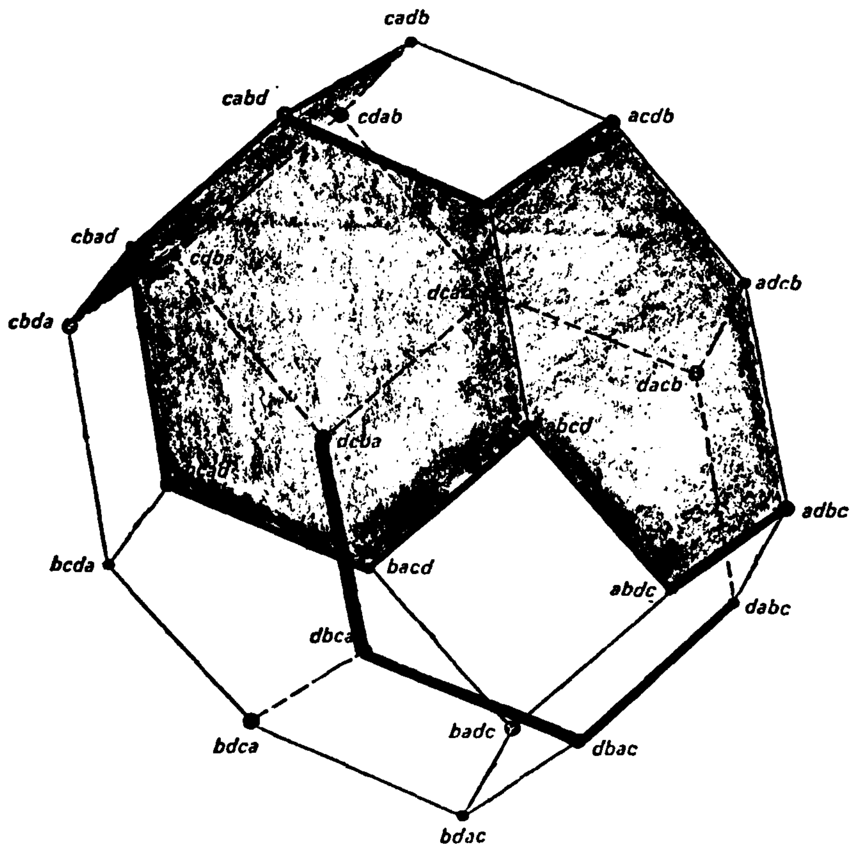

Figure 8

objects. Hence the only "not-in-second" condition will be on the fourth object (here b). The cyclic chain on which one can find the permutations where b is in the middle of at least one triple is not included in the three hexagons for four vertices (here d c b a, d b c a, d b a c, d a b c). But the three "not-in-second" conditions possible for b each imply the prohibition of two of these four vertices. This proves that the three extremal conditions have to prohibit no more than two hexagons.

1. If they are disjoint, i.e., if the three conditions are of type 1 or type 3, they will necessarily hold on two different objects x and y. Two of them will necessarily be x n 1/z t and y n 1/z t (or x n 3/z t and y n 3/z t)-- the third will add one of the four possible squares adjacent to only one hexagon only.

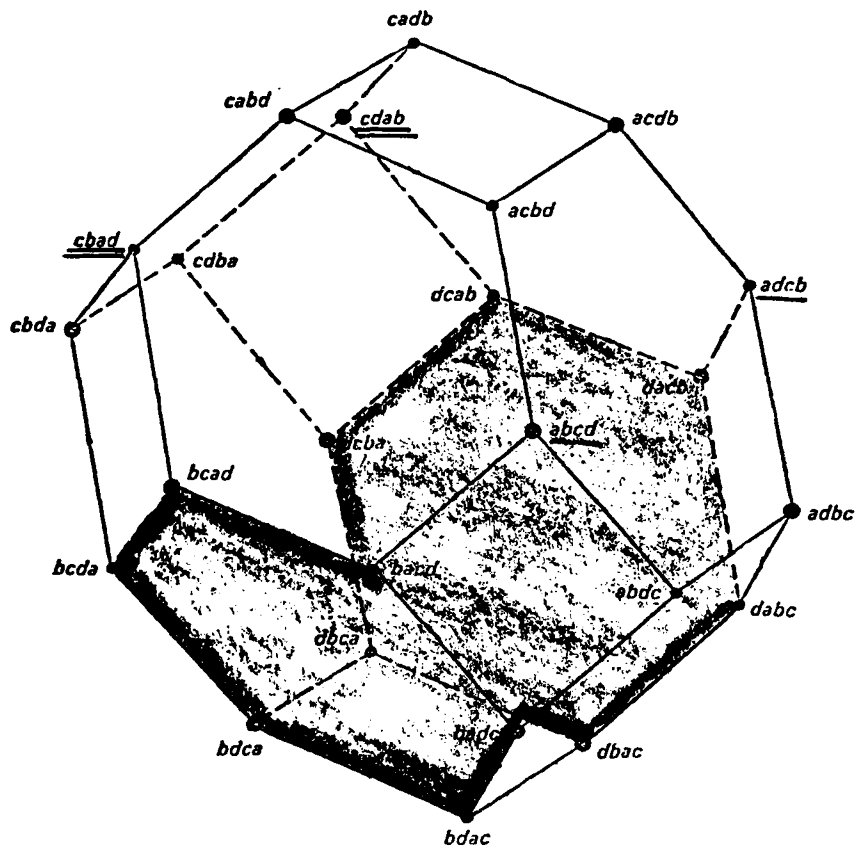

Figure 9

If one then looks to the representation shown in Figure 9 of the two cyclic lines representing the two possible remaining conditions of type 2, one clearly sees that, as they can only be z n 2/x y or t n 2/x y, one of only two different squares can be chosen. If z n 2/x y is chosen, the corresponding squares imply x or y n 1/{x,y,z} which is impossible, and if t n 2/x y is chosen, the corresponding squares imply x or y n 1/{x,y,t} which is impossible. (In Figure 9, one can use the correspondence b = x, d = y, a = x, c = t.)

2. If the two hexagons are joined by one edge, one can consider without loss of generality that two conditions will be of type 3 and one of type 1. Each of them will hold for different objects, just as it will be for the remaining condition of type 2. As shown in Figure 10, the part of the cyclic lines on which the vertices for the conditions of type 2 stand counts six vertices belonging to no hexagon. The squares added are necessarily at least two which raises the total of prohibited vertices to a minimum of 16.

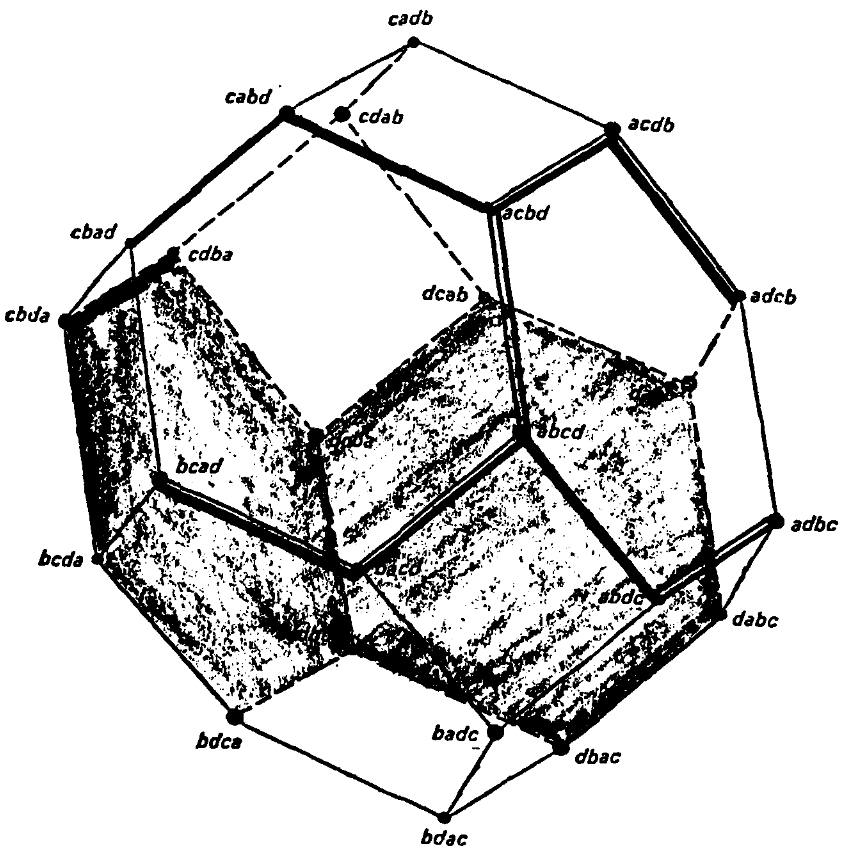

Figure 10

Finally, if the three extremal conditions use only one hexagon -- let it be for instance, as in Figure 11, d n 3/a b, d n 3/a c, d n 3/b c, -- the remaining condition of type two holds necessarily on the remaining triple {a,b,c}. That is to say, it representation on the permutohedron consists of two symmetric minimal chains, joining together corresponding elements of the hexagon "d is the first" and of the hexagon "d is the last."

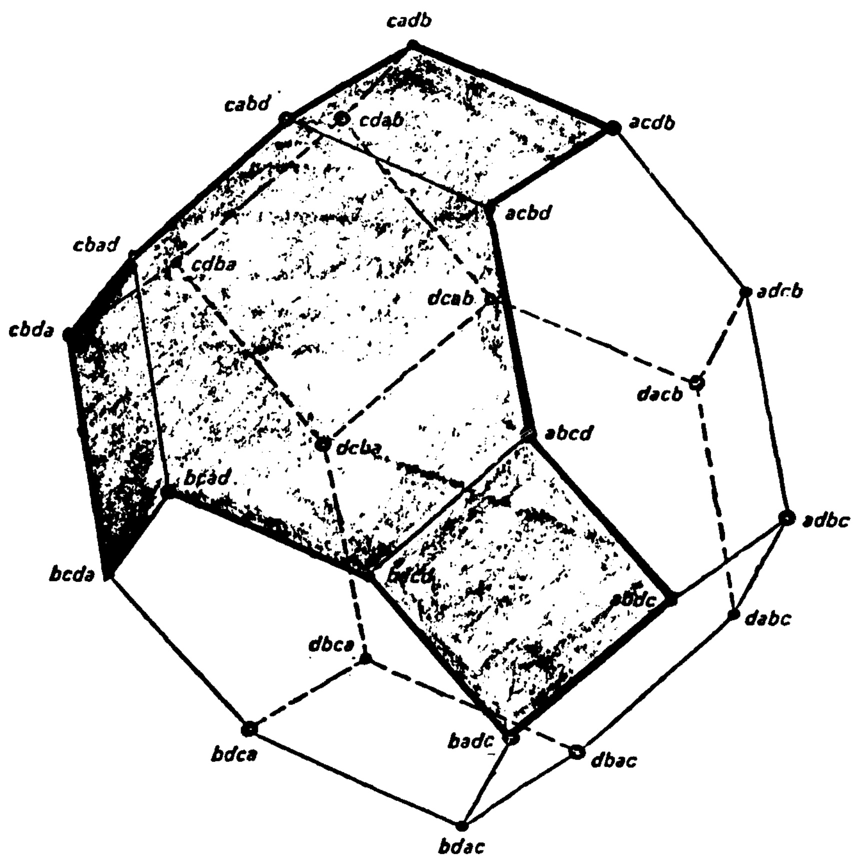

Figure 11

These conditions, necessarily add four new prohibited permutations to the twelve already forbidden ones.

Corollary 2: There is no candidate profile with exactly two conditions of type 2 (or two extremal conditions).

Proof:

1. The extremal conditions are obtained from only one hexagon. Without loss of generality, these conditions can be c n 3/a b and c n 3/d b, as in Figure 12, which prohibits ten vertices.

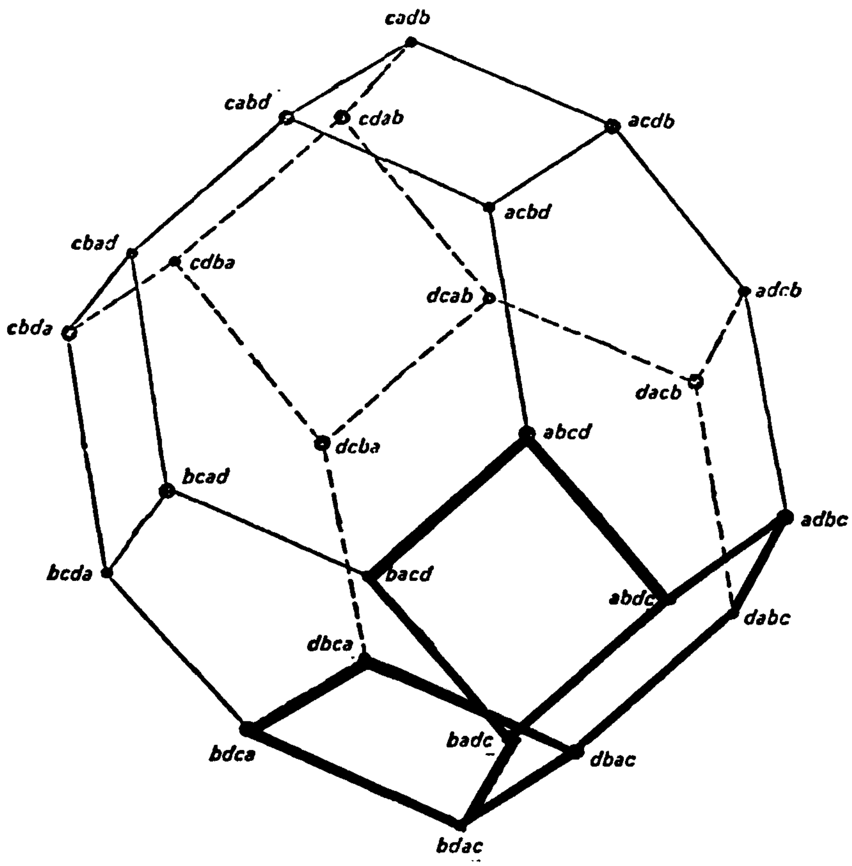

Figure 12

The remaining conditions are met:

-With any choice of two symmetric lines of Figure 13, because, for the triple {c,d,a}, only d and a can follow a condition of type 2 -- and this will add five prohibited vertices.

-With any choice of condition of type 2 on {a,b,d} -- (which clearly adds at least one prohibited vertex on the hexagon where c is first).

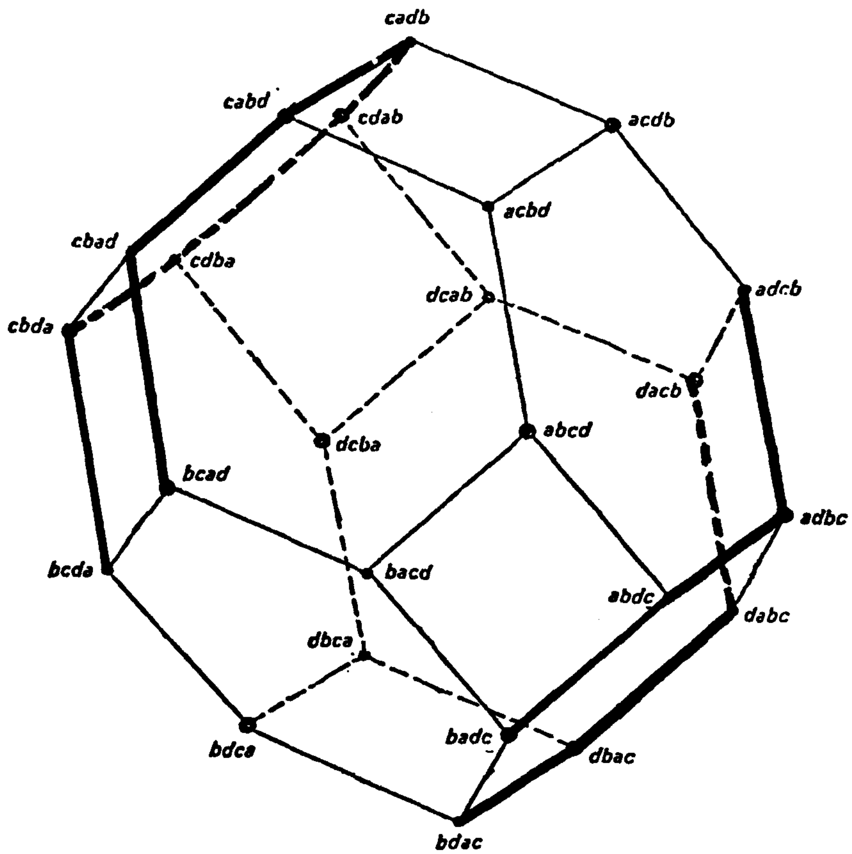

Figure 13

2. The extremal conditions can be obtained from two hexagons having one edge in common as in Figure 14.

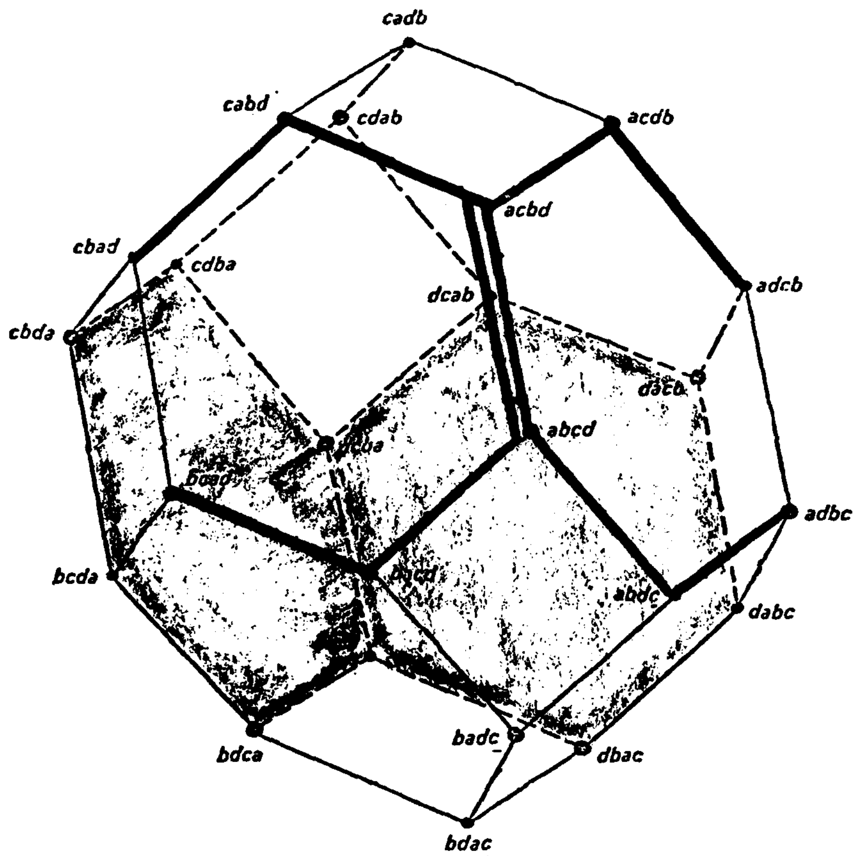

Figure 14

Then the two sets of four prohibited vertices for each condition of type 2 have to be taken along the bold lines. It is clear that the union of these sets contains at least six elements (which raises the total to 16).

3. The extremal conditions can be obtained from two disjoint hexagons as in Figure 15.

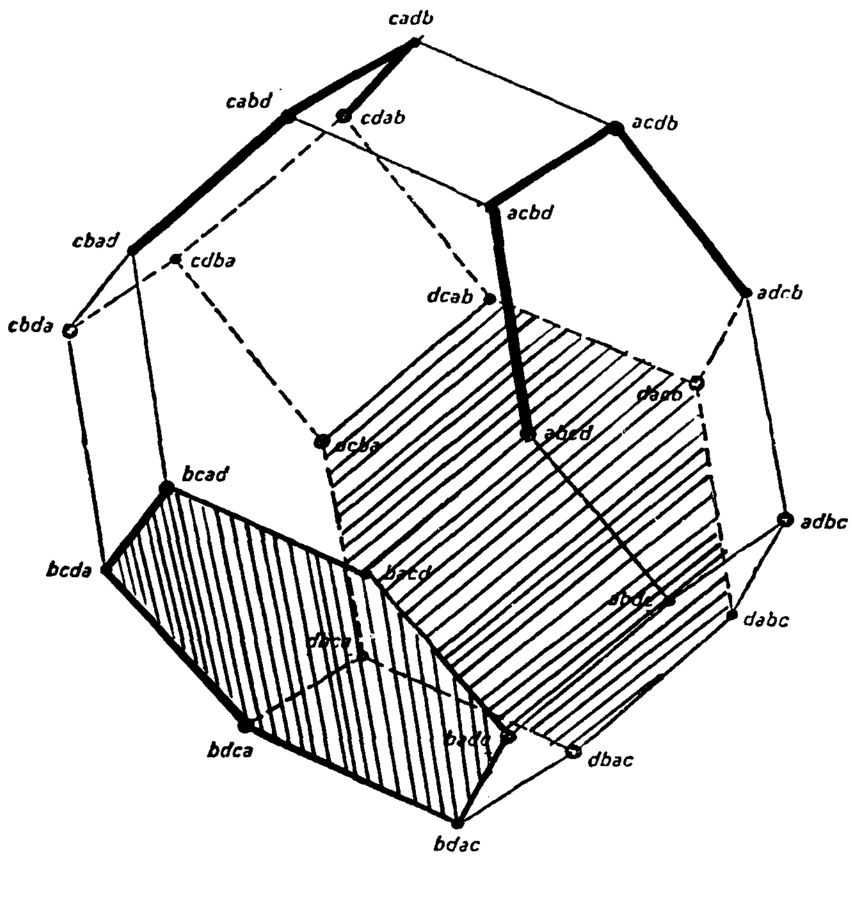

Figure 15

Hence, as the extremal conditions are on b and d, the conditions of type 2 have to be on a and c and their contribution, lying on the bold lines, consists of at least twice two vertices, and the profile cannot be a candidate.

Proposition 2: There is no candidate with conditions of type 2.

Proof: From Lemma 1 and Corollaries 1 and 2, it remains only to prove that there is no candidate profile with exactly three conditions of type 2.

Without loss of generality, one can suppose $c\,n\,3/a\,b$. Then one has necessarily three other objects in order to verify the three conditions of type 2. In particular, one should have:

- $a\,n\,2/b\,d$ or $a\,n\,2/c\,d$
- $b\,n\,2/a\,d$ or $b\,n\,2/c\,d$.

This is enough to prove the impossibility by the study of the three cases:

- $a\,n\,2/b\,d$ which implies $b\,n\,2/c\,d$
- $b\,n\,2/a\,d$ which implies $a\,n\,2/c\,d$
- $a\,n\,2/c\,d$ and $b\,n\,2/c\,d$.

They are described in Figures 16, 17, and 18 respectively. The conditions on a are above-lined, the conditions on b under-lined. In the first case, nine prohibited vertices are added; in each of the last two cases eight are added. Thus Proposition 2 holds.

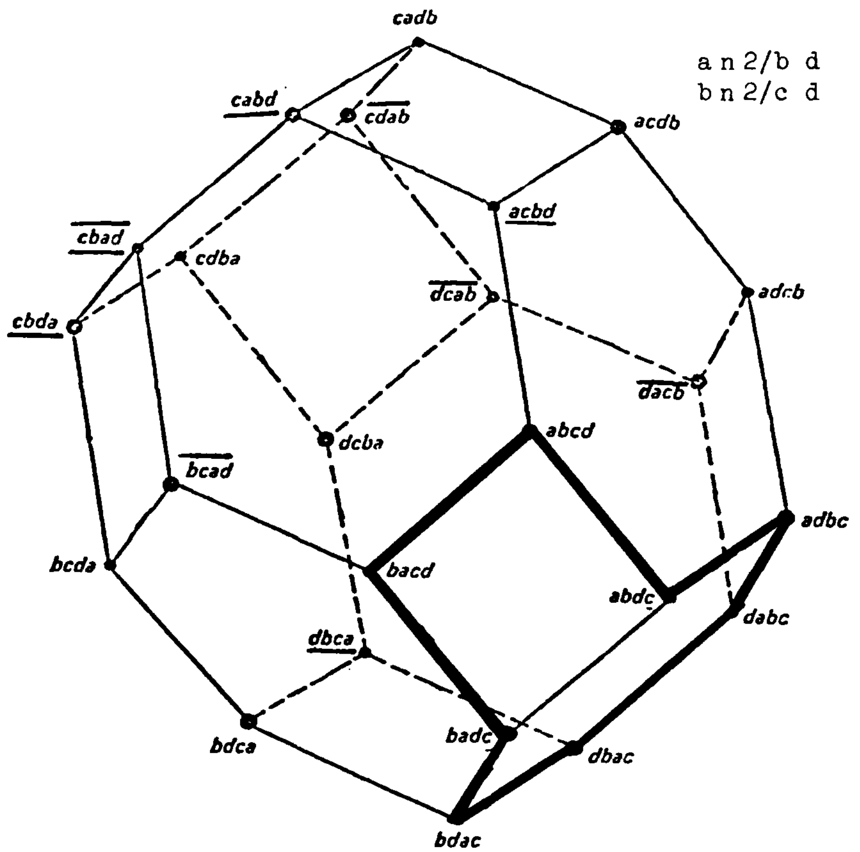

Figure 16

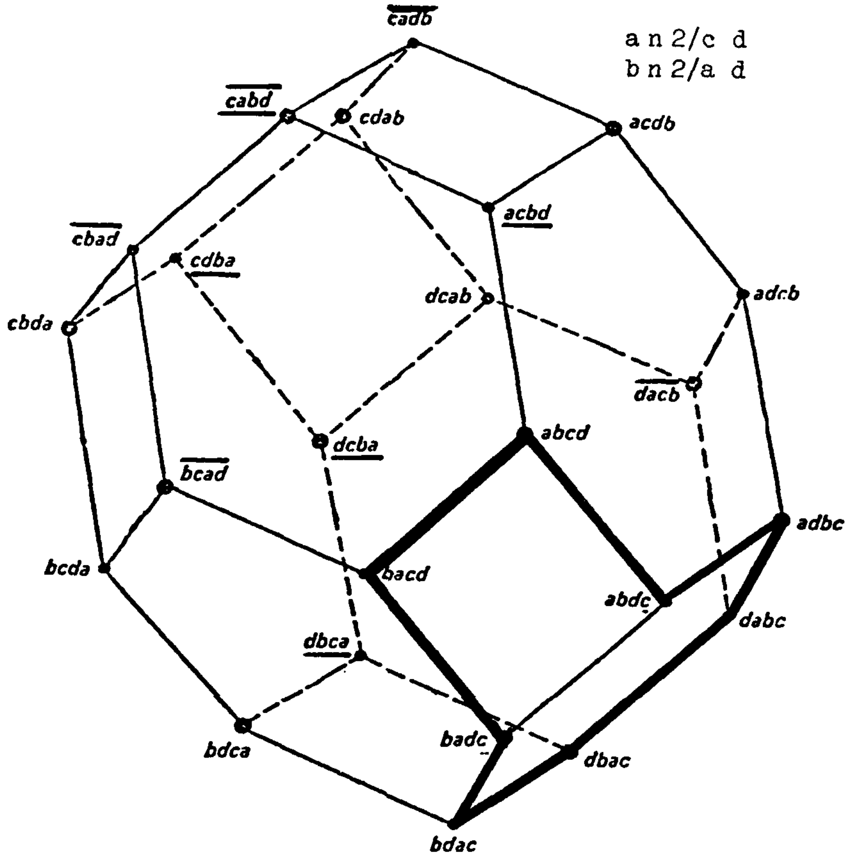
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Figure 17

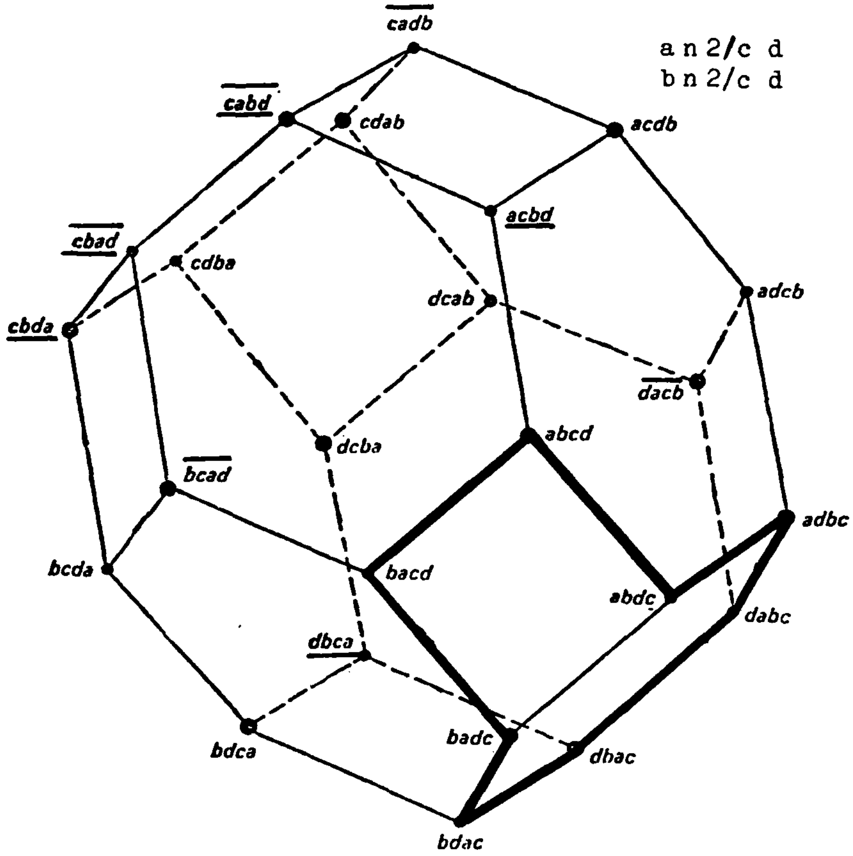
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Figure 18

III. We can now derive our final result.

Lemma 5: The objects on which the extremal conditions hold, in a candidate profile, cannot be more than two.

Proof: It is easy to see that the number of profiles containing four different and non-opposite hexagons are in very limited number, up to a rotation of the permutohedron. The two possible profiles, shown in Figures 19 and 20, exhibit more than 16 prohibited vertices.

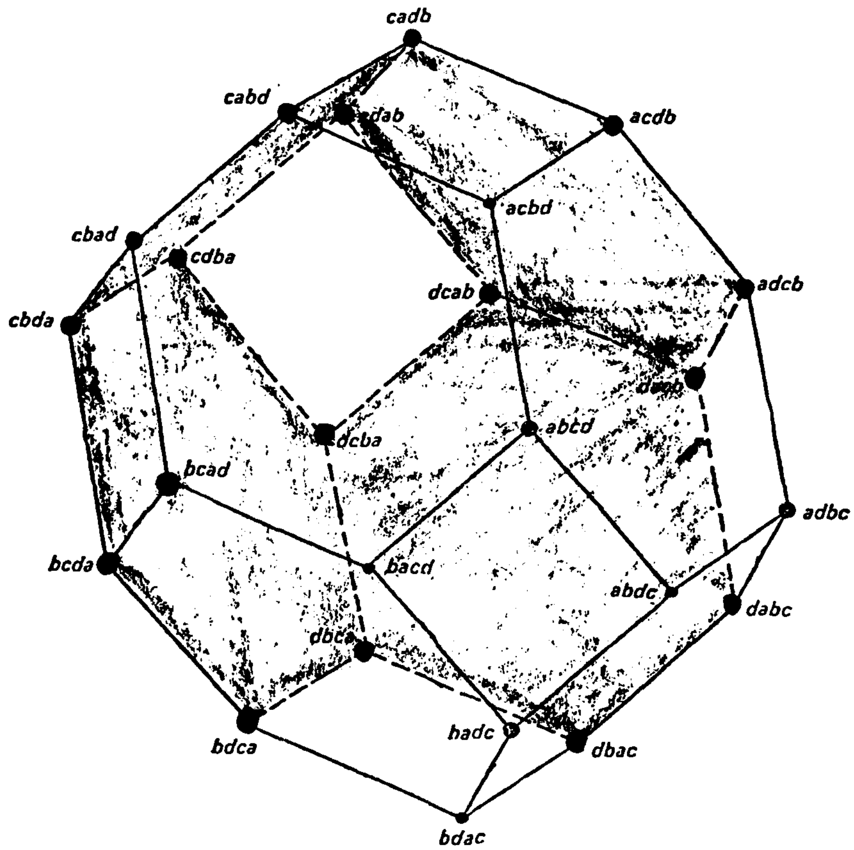


Figure 19

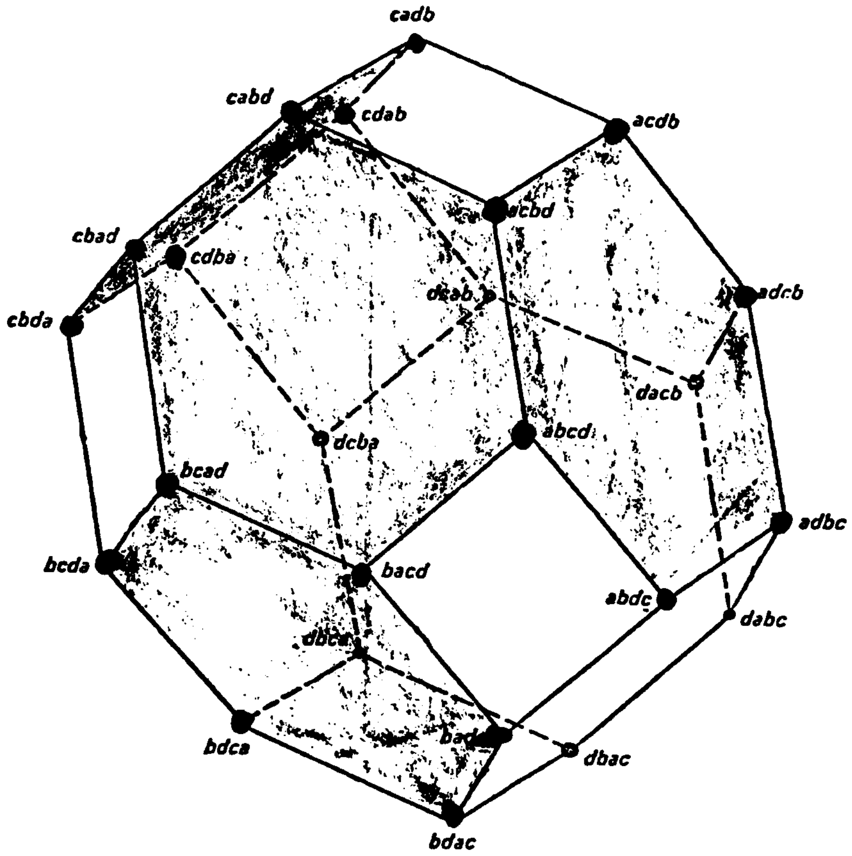


Figure 20

We have already seen, at the beginning of the proof of Corollary 1, that three hexagons have to share edges as in Figure 21 (14 prohibited vertices).

The three hexagons represented in this figure imply that:

1. The conditions on triples which would add no vertex would be b n 1/a c, d n 1/a c, a n 3/d b;
2. Those adding only one vertex would be a n 3/b c, b n 1/d a, a n 3/d c, d n 1/a b; none is on {b,c,d}.

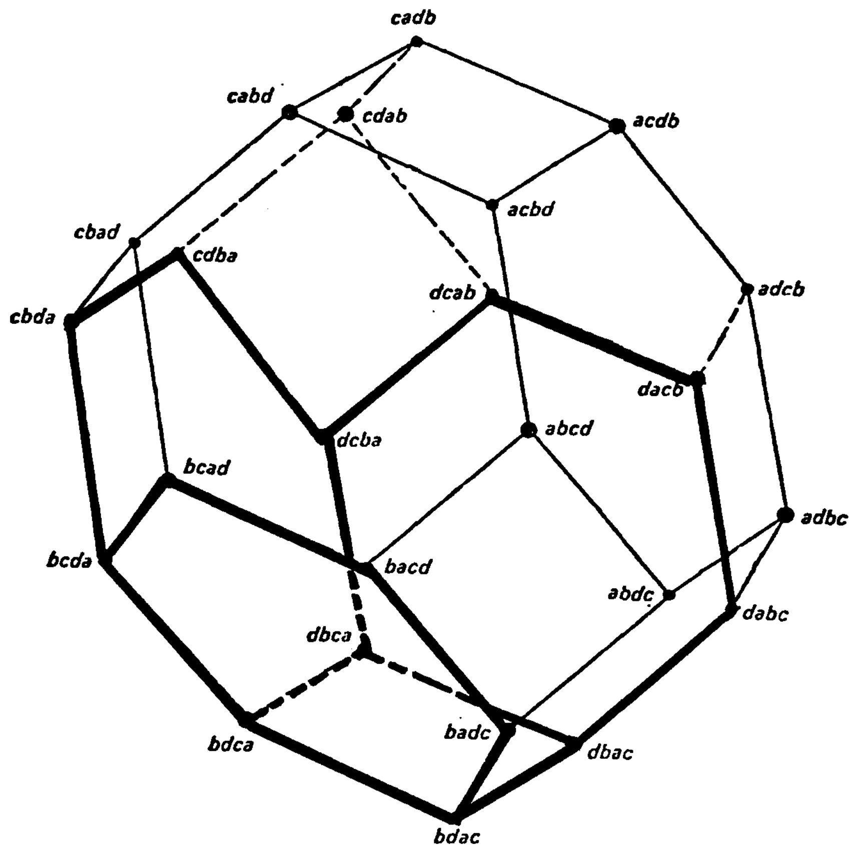


Figure 21

As the three-hexagons-profile cannot be a candidate, the lemma is proven.

Hence the candidate profiles need exactly two prohibited hexagons (as they imply four conditions and each hexagon at most three). These two hexagons cannot be opposite (Lemma 3). They cannot correspond to conditions of the same type because the necessarily added squares would add in turn at least four prohibited vertices.

Hence they hold on hexagons like those of Figure 22 for example.

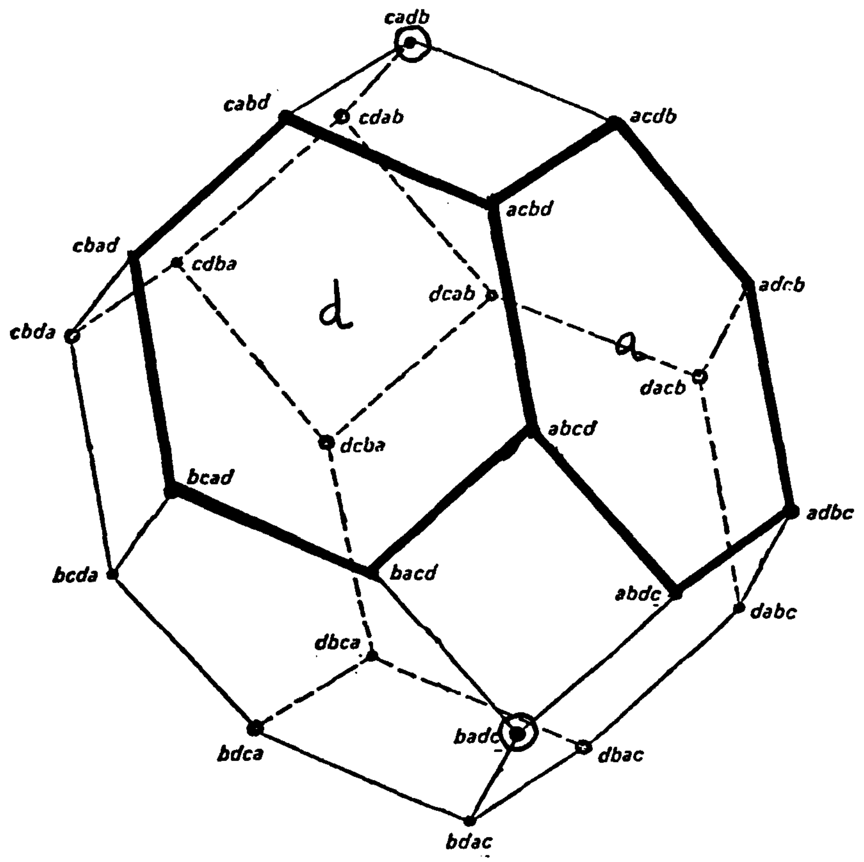


Figure 22

It is easy to see that if one hexagon, for example "d", was used in the three conditions d n 3, "a" would be used in the only condition a n 1 which holds on a triple without d and the total of 16 vertices would be reached.

The prohibition of vertices c a d b and b a d c implies, for the first one d n 3/c a and a n 1/b d for the second d n 3/a b and a n 1/c d. Hence at most one of these vertices, and the two squares

on the sides as in Figure 23 will be used, yielding a total number of 15 prohibited vertices.

These profiles are the only candidates.

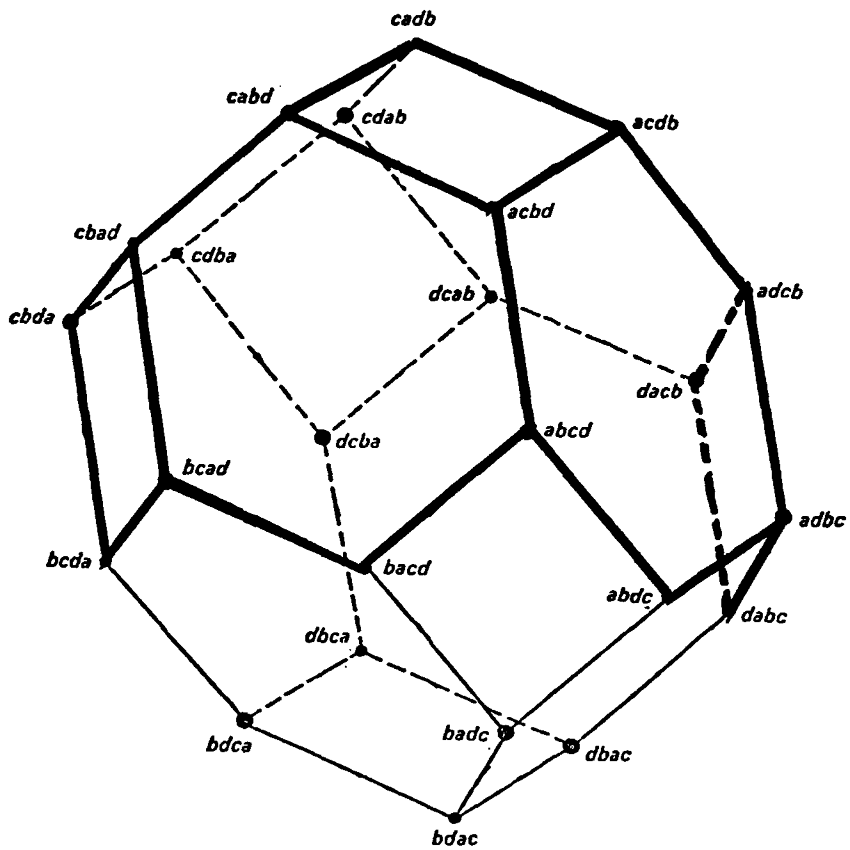
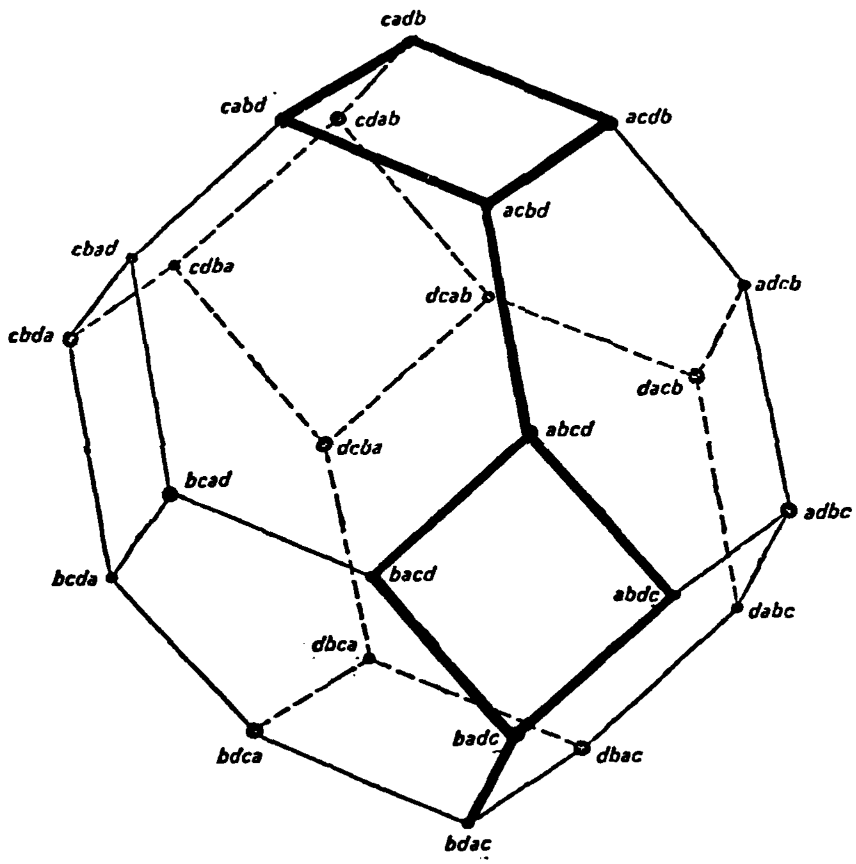


Figure 23

<u>Theorem</u>: The 16 profiles respecting the value restriction condition on four objects and containing the critical number of nine different votes have, up to an isomorphism, the following outline on the permutohedron (Figure 24):



<u>Figure 24</u>

Conclusion

We have thus obtained this very scarce structure which occurs only 24 times over the $\binom{24}{9}$ = 1, 307, 504.

This can explain why the way towards this figure was necessarily somewhat painful.

The existence of the configuration with nine vertices on four objects has been shown to me by E. Terrier, then one of my Ph.D. students; J.C. Ayel (University of Savoie, Chambery) confirmed its unicity by use of a computer.

J. Ayel and J.C. Ayel found by computer enumeration for five alternatives that the maximal configuration was counting 20 vertices interpretation of the critical structure in terms of voting theory is not easy. It perhaps means that the maxima of $M(n)$, in the general case, are rare and very steep, and for these reasons, do not have a great practical interest. In any case the scarceness of the maximal profiles lets one think that the value restriction conditions are more and more restricting in terms of individual and collective freedom as the number of alternatives increases.

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